Modeling Stochastic Behaviors of Packet Pair Technique

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Abstract

In this paper, we give a stochastic analysis on the packet pair technique for available bandwidth measurement and derive an explicit delay model which is accurate enough to capture the stochastic nature of cross traffic. We study the single-hop case and derive an approximate model, which explicitly shows the functional relationship between the input and the output gaps of a packet pair, with the link capacity and the moments of cross traffic as parameters. We also examine the multi-hop case and derive an explicit multi-hop model under an assumption of a single tight link. We validate our model via \texttt{ns}-2 simulations and empirical results. Based on the proposed model, we give some important insights into measurement of available bandwidth. We point out that the stochastic cross traffic can introduce significant error in available bandwidth estimation. By using the concepts of the characteristic value and the critical value, we show that the effect of stochastic cross traffic is most severe when the input probing gap is around the characteristic value. In addition, we discuss the influence of stochastic cross traffic on several available bandwidth measurement algorithms.

Keywords

Packet pair technique, end-to-end available bandwidth measurement, approximate stochastic model, Poisson process
I. Introduction

To improve the performance of Internet applications in various aspects, it is crucial to understand and exploit properties of an end-to-end path. Among the many attributes of an end-to-end path, the available bandwidth is one of the most useful characteristics, which can be used in numerous ways [1] [2]. For example, in many bandwidth-sensitive applications such as peer-to-peer applications and on-demand multimedia streaming applications, a client will benefit if it is connected to a peer via a path with a sufficient available bandwidth. Further, the performance of overlay networks can be improved if the available bandwidth between nodes can be estimated.

However, it is a challenging research task to design a mechanism that can accurately measure the end-to-end available bandwidth. A general way of active measurement is to inject probing packets into an end-to-end path and observe their behavior to estimate path characteristics of interest. One of the most popular mechanisms is the packet pair technique [3] [4], in which a source sends multiple packet pairs to a receiver. Each packet pair consists of two probing packets usually of the same size. The inter-packet dispersion between these two probing packets changes according to path characteristics such as link capacities and cross traffic. Hence, the inter-packet dispersion of two probing packets at the sender and that at the receiver is exploited to estimate the end-to-end available bandwidth.

The packet pair technique has been extensively studied for bandwidth measurement. Actually, many tools for measuring the end-to-end bottleneck capacity and the end-to-end available bandwidth are related to the packet pair technique [1] [2] [5] [6] [7] [8] [9] [10] [11] [12] [13]. Most of these measurement methods rely on qualitative aspects instead of an explicit mathematical model. Recently, a deterministic packet pair model has been developed in [9] [10] [11]. However, this deterministic model regards cross traffic as fluid with a constant rate and ignores its stochastic nature, i.e., the burstiness of cross traffic. More recently, Liu et al. [14] has given a stochastic analysis on packet pair/train probing. In
[14], an asymptotic behavior of packet pair/probing has been investigated by a sample-path analysis and it has been discovered that there is a bias in available bandwidth estimation due to stochastic cross traffic.

The main contribution of this paper is a derivation of an explicit model for the packet pair technique, which is accurate enough to capture the stochastic nature of cross traffic. In [14], a sample-path analysis on single-hop packet pair/train probing has been successfully given under mild assumptions and some upper and lower bounds were given. Here, we derive a simple formula that shows an explicit mathematical relationship between the input and the output probing gaps. We investigate the single-hop case and derive an approximate model, which explicitly shows the functional relationship between the input and the output gaps of a packet pair, with the link capacity and the moments of cross traffic as parameters. We also examine the multi-hop case and derive an explicit multi-hop model for the case of a single tight link. We validate our model via ns-2 simulations and empirical results. Then, based on the proposed model, we discuss the impact of stochastic cross traffic on available bandwidth measurement. By using the concepts of the characteristic value and the critical value, we explain the reason why it is difficult to estimate the available bandwidth accurately. Then, we discuss the effect of stochastic cross traffic on several measurement algorithms.

The rest of the paper is organized as follows: In Section 2, we provide preliminaries for the problem addressed in the paper. In Section 3 and 4, we introduce a stochastic model, which can capture the essential stochastic nature of cross traffic. In Section 5, we validate the stochastic model through ns-2 simulations and empirical results. Based on the model, we discuss the impact of stochastic cross traffic on the available bandwidth measurement in Section 6. Finally, the conclusion follows in Section 7.
II. Preliminaries

In this section, we introduce the deterministic packet pair model [9] [10] [11]. The model describes the relationship between the inter-departure time of two probing packets of a packet pair at a sender and the inter-arrival time at a receiver. In [9] and [11], a single-hop model was proposed, while a multi-hop model was also derived in [10]. However, all in [9] [10] [11], cross traffic was assumed fluid with a constant rate and the stochastic nature of cross traffic was ignored.

The packet pair technique works as follows: A source sends packet pairs to a receiver. Each packet pair consists of two probing packets, usually of the same size. The time dispersion between these two packets changes according to path characteristics such as link capacities and cross traffic as they traverse the path. Hence, the difference between the inter-departure times of two packets at the sender and the inter-arrival times at the receiver is exploited to obtain the information on the available bandwidth. The inter-departure time $\Delta_{\text{in}}$ of a packet pair is defined as the time interval between the instant when the last bit of the first packet leaves the sender and that when the last bit of the second packet leaves the sender. In a similar manner, the inter-arrival time $\Delta_{\text{out}}$ of a packet pair is defined as the time interval between the instant when the last bit of the first probing packet arrives at the receiver and that when the last bit of the second probing packet arrives at the receiver. Our main concern here is how to establish a more accurate mathematical relationship between $\Delta_{\text{out}}$ and $\Delta_{\text{in}}$ than the deterministic model in [9] [10] [11].

Consider a single link with capacity $C$. The sender at one end of the link transmits two probing packets to the receiver at the other end. The size of each probing packet is $L_p$ bytes. Let $q$ denote the amount of traffic queued when the first probing packet arrives at the queue. Figure 1 illustrates the arrival and departure of a packet pair on a single link. In the figure, $T$ denotes the time interval between the instant when the last bit of the first probing packet arrives at the receiver and the instant when the last bit of the second probing
packet departs from the receiver, and $X(\Delta_{in})$ denotes the amount of cross traffic arrived during $\Delta_{in}$. The server is either busy or idle at the arrival instant of the second probing packet. The case in which the second probing packet sees the server busy is given in Fig. 1 (a). Note that we illustrate the full utilization case as a representative for a busy server in Fig. 1 (a). Similarly, the case of an idle server is presented in Fig. 1 (b). In general, it should be noted that it is the lastest busy period that determines whether the server is busy or idle when the second probing packet arrives. If the lastest busy period ends before the second probing packet arrives, the server will be idle at the arrival instant of the second probing packet and vice versa.

When cross traffic is fluid with a constant rate $r$, $X(\Delta_{in})$ becomes a deterministic value of $r\Delta_{in}$ and the following relationship between $\Delta_{out}$ and $\Delta_{in}$ was derived depending on whether the link is fully utilized or not [10], [11]:

$$
\Delta_{out} = \begin{cases} 
\frac{C}{r}\Delta_{in} + \frac{L_p}{C}, & \Delta_{in} \leq \Delta^{*} (= \frac{L_p + q}{C - r}); \\
\Delta_{in} - \frac{q}{C}, & \text{otherwise}. 
\end{cases}
$$

From (1), we know that the graph of $(\Delta_{in}, \Delta_{out})$ consists of two line segments and the change of slope occurs at $\Delta_{in} = \Delta^{*} (= \frac{L_p + q}{C - r})$, which is termed as the characteristic value [10]. Let $\Delta^{c}$ denote the value of $\Delta_{in}$ such that $\Delta_{in} = \Delta_{out}$, then we have $\Delta^{c} = \frac{L_p}{C}$. Here, we call $\Delta^{c}$ the critical value. Note that we can obtain the available bandwidth $A$ once we know $\Delta^{c}$ by $A = C - r = L_p/\Delta^{c}$. If $q$ is negligible, then $\Delta^{c} \approx \Delta^{*}$ and we have $C - r \approx L_p/\Delta^{*}$.

Many measurement algorithms infer the available bandwidth by using an estimate of $\Delta^{*}$ [2] [8] [11] [13]. Since cross traffic is stochastic, we can easily expect that considerable amounts of modeling error is introduced in (1) by the assumption that cross traffic is fluid with a constant rate. In this paper, we consider $X(\Delta_{in})$ as a stochastic process and derive a stochastic model that depicts the relationship between $\Delta_{out}$ and $\Delta_{in}$ more accurately. Based on the model, we investigate how the available bandwidth measurement is influenced by the stochastic nature of cross traffic.
III. Stochastic packet-pair model: Single-hop case

In this section, our primary concern is to derive a single-hop relationship between the input probing gap and the output probing gap under an assumption of the stationary Poisson cross traffic. First, we derive an explicit relationship between the input probing gap and the output probing gap under the Poisson cross traffic with the same packet size. Then, we consider the case of the Poisson cross traffic with various packet sizes. In addition, we examine the case of the aggregated ON-OFF cross traffic to study how the self-similarity of cross traffic influences the model.

A. Poisson cross traffic with the same packet size

Here, we derive a stochastic packet pair model based on an assumption that cross traffic is a stationary Poisson process in the time scale of interest. Recent research has revealed that Internet traffic can be considered as a Poisson process in sub-second time scales [15] [16]. Also, the stationary assumption on the Internet traffic is shown to be empirically acceptable over intervals of several minutes up to a few hours [16] [17].

Consider a single-hop link as an M/D/1 queue. We tentatively assume that all the packet sizes of cross traffic are the same with $L_0$ bytes. This assumption will be relaxed later. First, we derive a formula for the average amount of traffic queued when the first probing packet arrives at the queue. We denote the queue size when the first probing packet arrives at the queue as $q$ [bytes]. Further, we denote the average packet arrival rate [packets/s] and the link capacity [bytes/s] as $\lambda$ and $C$, respectively. Then, the average traffic rate [bytes/s] and the link utilization are $r = L_0 \lambda$ and $u = r/C$, respectively. The Pollaczek-Khintchine (PK) formula [18] gives the following result on the average queue size :

$$\bar{q} = \mathbb{E}[q] = \frac{u^2}{2(1-u)} \quad (2)$$

Now, we derive a stochastic relationship between $\Delta_{out}$ and $\Delta_{in}$. Let $L_p$ and $N(t)$ denote the probing packet size in bytes and the number of cross packets arrived in $(0,t)$, respectively.
Let $A_k$ denote the event of arriving $k$ cross packets in $(t_0, t_0 + \Delta_{in}]$ where $t_0$ is the instant when the last bit of the first packet of a packet pair arrives at the queue. Then, from the nature of the Poisson traffic, $A_k$ becomes the event of $N(\Delta_{in}) = k$.

One of the crucial differences between the stochastic and the deterministic cross traffic is the amount of traffic arrived during $(t_0, t_0 + \Delta_{in}]$. In the stochastic framework, the amount of cross traffic is a random process with its probability distribution while that in the deterministic setting is constant. However, consideration only on the amount of traffic during $(t_0, t_0 + \Delta_{in}]$ may not be sufficient to fully reflect the stochastic nature of cross traffic. For example, even when $\Delta_{in} \gg L_0/C$ and only one cross packet has arrived during $(t_0, t_0 + \Delta_{in}]$, the second probing packet can still wait for service if the cross packet arrives at $t = t_0 + \Delta_{in} - \epsilon$ where $0 < \epsilon < L_0/C$. Hence, we also need to consider the arrival instants of cross traffic packets.

However, developing a model which takes account of both the amount of cross traffic and the arrival instants of cross traffic packets will introduce a great deal of complexity. In fact, such a model will require the transient analysis of $G/D/1$ queue that is known to be extremely complex [18]. Hence, we develop an approximate model which is accurate enough to capture the stochastic nature of cross traffic.

The key idea is to use the intrinsic property of the Poisson traffic, i.e, at most one packet arrives in a very small time interval $\Delta t$. Consider the interval of $\Delta_{in}$ slotted with $\Delta t = L_0/C$. In fact, the size of the first slot is set to $(L_p + L_0)/C$ so that $\Delta_{in}$ consists of $[(C\Delta_{in} - L_p)/L_0]$ slots. However, we ignore this since it does not make much difference in analysis. Since $\Delta t$ is very small for large $C$, we have

$$N(\Delta t) = \begin{cases} 1, & \text{with probability } \lambda \Delta t + o(\Delta t), \\ 0, & \text{with probability } 1 - \lambda \Delta t - o(\Delta t). \end{cases}$$ (3)

Consider the collection of all the events, $\{A_k | k = 0, \ldots, \infty\}$, which is denoted by $\Sigma$. Also, let $P_k$ denote the probability of $A_k$. Let $k^*$ be the smallest $k$ such that $k = N(\Delta_{in}) \geq$
$(C\Delta_{in} - L_p)/L_0 - q$ at $t = t_0$. However, we ignore $q$ since $q \ll (C\Delta_{in} - L_p)/L_0$ and this makes the derivation more tractable. Since $k \geq 0$, we have $k^* = \max(0, [(C\Delta_{in} - L_p)/L_0])$, which corresponds to the total number of slots in $\Delta_{in}$. Let $\Gamma = \{A_k| k = k^*, \cdots, \infty\}$. For a given $A_k$, let $B_n$ denote the event of $n = (n_i, i = 1, \cdots, k^*)$ where $n_i$ is the number of packets arrived in the $i$-th slot. Note that $n \in N$ where $N := \{n| \forall n_i \geq 0, \sum_{i=1}^{k^*} n_i = k\}$. With (3), we have $\text{Prob( server idle at } t = t_0 + \Delta_{in}|A_k) \gg \text{Prob( server busy at } t = t_0 + \Delta_{in}|A_k)$ for $k < k^*$, i.e., $\text{Prob( server idle at } t = t_0 + \Delta_{in}|A_k) \approx 1$. This implies that the second probing packet almost always sees the server idle when $k < k^*$. In a similar way, for $k \geq k^*$, $$\sum_{n\in N \setminus N_0} \text{Prob}(B_n|A_k) \gg \sum_{n\in N_0} \text{Prob}(B_n|A_k)$$ where $N_0 := \{n = (n_i, i = 1, \cdots, k^*)|n \in N, \exists j\text{ such that } n_j = 0\}$. Consequently, $\sum_{n\in N \setminus N_0} \text{Prob}(B_n|A_k) \approx 1$ for $k \geq k^*$. With the above approximations, $\Delta_{out}$ becomes a random variable with the following distribution:

$$\Delta_{out} = \left\{ \begin{array}{ll}
\frac{kL_0 + L_p}{C}, & \text{with } P_k, k = k^*, k^* + 1, k^* + 2, \cdots \\
\Delta_{in} - \frac{q}{C}, & \text{with } P(\Gamma^c) = 1 - \sum_{k=k^*}^{\infty} P_k.
\end{array} \right.$$ 

Note that $q$ is also a random process, with the mean given in (2). The expected value of $\Delta_{out}$ becomes

$$\mathbb{E}[\Delta_{out}] = \sum_{k=k^*}^{\infty} \left( \frac{kL_0 + L_p}{C} \right) P_k + \left( \Delta_{in} - \frac{q}{C} \right) P(\Gamma^c).$$

Note that $P_k$ for the Poisson process with the mean arrival rate $\lambda$ is

$$P_k = e^{-\lambda\Delta_{in}} \frac{(\lambda \Delta_{in})^k}{k!}.$$  

(4)
By using (4), we have

\[ \Delta_{\text{out}} = \frac{1}{C} \left[ L_0 \lambda \Delta_{\text{in}} \sum_{k=k^*-1}^{\infty} P_k + L_p \sum_{k=k^*}^{\infty} P_k \right] \]

\[ + (\Delta_{\text{in}} - \frac{\eta}{C}) P(\Gamma^c) \]

\[ = P(\Gamma) \left[ (u-1) \Delta_{\text{in}} + \frac{L_p + \eta}{C} \right] \]

\[ + u P_{k^*-1} + 1) \Delta_{\text{in}} - \frac{\eta}{C}. \]  

(5)

We can see that \( \Delta_{\text{out}} \) is a function of \( \Delta_{\text{in}} \) and the parameters such as the link capacity and the cross traffic statistics. However, (5) is not sufficient to show the explicit relationship between \( \Delta_{\text{out}} \) and \( \Delta_{\text{in}} \) because of \( P(\Gamma) \), which is also a function of \( \Delta_{\text{in}} \). Hence, we use the Gaussian approximation for the Poisson cross traffic [19]. The cumulative density function of the normalized Gaussian distribution is expressed as

\[ F_Z(z) = 1 - Q(z), \]

where

\[ Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^{\infty} e^{-\xi^2/2} d\xi. \]

Now, we have

\[ P(\Gamma) = \sum_{k=k^*}^{\infty} P_k \approx Q(z_{k^*}), \]

where \( z_{k^*} = \frac{k^*-\lambda \Delta_{\text{in}}}{\sqrt{\lambda \Delta_{\text{in}}}} = \frac{(C/L_0-\lambda) \Delta_{\text{in}} - L_p/L_0}{\sqrt{\lambda \Delta_{\text{in}}}} \).

From \( P_{k^*-1} \approx Q(z_{k^*-1}) - Q(z_{k^*}) \), we have the following relationship between \( \Delta_{\text{in}} \) and \( \Delta_{\text{out}} \):

\[ \Delta_{\text{out}} \approx \left( u Q(z_{k^*-1}) - Q(z_{k^*}) + 1 \right) \Delta_{\text{in}} \]

\[ + \frac{1}{C} (L_p Q(z_{k^*}) - (1 - Q(z_{k^*})) \eta). \]  

(6)

Remark. The deterministic model (1) can be regarded as an asymptote of (6). When \( \Delta_{\text{in}} \to 0 \), both \( Q(z_{k^*}) \) and \( Q(z_{k^*-1}) \) approach 1. Then (6) becomes \( \Delta_{\text{out}} = u \Delta_{\text{in}} + \frac{L_p}{C} \).
This agrees with the deterministic model when $\Delta_{in} \leq \Delta^*$. When $\Delta_{in} \to \infty$, both $Q(z_k^*)$ and $Q(z_k^*-1)$ approach 0, and (6) becomes $\overline{\Sigma}_{out} = \Delta_{in} - \frac{q}{c}$, which corresponds to the deterministic case when $\Delta_{in} \geq \Delta^*$. Hence, we can see that (6) is a generalization of (1).

B. Poisson cross traffic with various packet sizes

In the above discussions, we have assumed that the packet size of cross traffic is fixed, i.e., $L_0$ takes only one value. However, this is not true in real network situation. For a more realistic model, we consider the case of cross traffic with various packet sizes. Here, we adopt a discrete distribution of the packet size in cross traffic and assume that the packet size of each flow takes one of $n$ values. Let $L_i$ [bytes], $\lambda_i$ [packets/s], and $N_i(t)$ [packets] denote the packet size, sending rate, and number of packets during $(0, t]$ for the $i$-th flow of cross traffic, respectively.

Then, the total traffic arrived during $\Delta_{in}$ is

$$X(\Delta_{in}) = \sum_{i=1}^{n} L_i N_i(\Delta_{in}) \text{ [bytes]}.$$ 

Here, we assume that the $i$th traffic flow is an independent Poisson process with $r_i$, $i = 1, 2, \cdots, n$. Let $A_k, k = (k_1, \cdots, k_n)$ denote the event of arriving $k_i$ packets for each flow $i$, $i = 1, 2, \cdots, n$ during the time interval of $[t_0, t_0+\Delta_{in}]$. For $\mathbf{N}(\Delta_{in}) := (N_1(\Delta_{in}), \cdots, N_n(\Delta_{in}))$, $A_k$ becomes the event of $\mathbf{N}(\Delta_{in}) = k$ as in the case of the single packet size.

Now, we consider a collection of all the possible events, $\{A_k \mid k \succeq 0\} =: \Sigma$, where $\succeq$ denotes the componentwise inequality. Let $I(\mathbf{N}(\Delta_{in})) := X(\Delta_{in}) - (C\Delta_{in} - L_p)$ and $\Omega_I := \{k \mid I(k) = X_k - (C\Delta_{in} - L_p) \geq 0\}$ where $X_k := X(\Delta_{in})|_{\mathbf{N}(\Delta_{in})=k}$. Also, let $\Gamma_I$ denote the collection of all the events $A_k, k \in \Omega_I$, i.e., $\Gamma_I = \{A_k \mid k \in \Omega_I\}$. Then we have

$$\Delta_{out} = \begin{cases} \frac{X_k + L_p}{c} & \text{with } P_k, k \in \Omega_I, \\ \Delta_{in} - \frac{q}{c} & \text{with } P(\Gamma_I^c) = 1 - \sum_{k \in \Omega_I} P_k. \end{cases}$$
Hence, $\Sigma_{out}$ becomes
\[
\Sigma_{out} = \sum_{k \in \Omega_I} \frac{X_k + L_p P_k}{C} + \left( \Delta_{in} - \frac{q}{C} \right) \left( 1 - P(\Gamma_I) \right)
\]
\[
= \frac{1}{C} \sum_{k \in \Omega_I} X_k P_k - \left( \Delta_{in} - \frac{L_p + q}{C} \right) P(\Gamma_I)
\]
\[
+ \left( \Delta_{in} - \frac{q}{C} \right).
\]

As in the previous section, we use the Gaussian approximation for \(X(\Delta_{in})\). Let \(X(\Delta_{in}) \sim \mathcal{N}(m, \sigma^2)\) where \(m = (\sum_i L_i \lambda_i) \Delta_{in}\) and \(\sigma^2 = (\sum_i L_i^2 \lambda_i) \Delta_{in}\), respectively. Using the relation \(df_X(x)/dx = -\frac{(x - m)}{\sigma^2} f_X(x)\) for the Gaussian pdf \(f_X(x)\),
\[
\sum_{k \in \Omega_I} X_k P_k \approx \int_{x^*}^{\infty} x f_X(x) dx
\]
\[
= \sigma^2 f_X(x^*) + m \int_{x^*}^{\infty} f_X(x) dx,
\]
where \(x^* = \max(0, [C \Delta_{in} - L_p])\). With the normalized Gaussian variable \(Z \sim \mathcal{N}(0, 1)\), we can further express (7) as
\[
\sum_{k \in \Omega_I} X_k P_k = \sigma f_Z(z^*) + m \int_{z_k^*}^{\infty} z f_Z(z) dz
\]
\[
= \sigma f_Z(z^*) + m Q(z^*),
\]
where \(z^* = (x^* - m)/\sigma\). For \(x' = \max(0, [C \Delta_{in} - L_p - 1])\), \(f_Z(z^*) \approx (F(z^*) - F(z')) / (z^* - z') = \sigma(Q(z') - Q(z^*))\) where \(z' = (x' - m)/\sigma\). Then, we have
\[
\sum_{k \in \Omega_I} X_k P_k = \sigma^2 (Q(z') - Q(z^*)) + m Q(z^*)
\]
\[
= \left[ K_\sigma (Q(z') - Q(z^*)) + K_m Q(z^*) \right] \Delta_{in},
\]
where \(K_m = \sum_{i=1}^n L_i \lambda_i\), \(K_\sigma = \sum_{i=1}^n L_i^2 \lambda_i\).
Hence, we have the following relationship between $\Delta_{out}$ and $\Delta_{in}$:

\[
\Delta_{out} = \frac{1}{C} \left[ K_\sigma (Q(z') - Q(z^*)) + K_m Q(z^*) \right] \Delta_{in} \\
- \left( \Delta_{in} - \frac{L_p + \bar{q}}{C} \right) Q(z^*) + \left( \Delta_{in} - \bar{q} \right) \\
= \left[ \frac{K_\sigma (Q(z') - Q(z^*)) + K_m Q(z^*)}{C} - Q(z^*) + 1 \right] \\
\Delta_{in} + \frac{1}{C} \left[ L_p Q(z^*) - (1 - Q(z^*))\bar{q} \right] \\
\] (8)

**Remark.** If we compare (8) with (6) for $n = 1$, we can notice that they are not equal. The reason is that we have introduced the Gaussian approximation in the first place in (8) while, in (6), we use the Poisson probability first and then, apply the Gaussian approximation. However, for $n = 1$, $Q(z^*) \approx 1/2$ when $\Delta_{in}$ is around $\Delta^*$, we have $K_\sigma f_X(x^*) \leq L_1 \sqrt{\lambda_1 \Delta_{in}}/2\pi \ll L_1 \lambda_1 Q(z^*) = K_m Q(z^*)$, and we can further approximate (8) with

\[
\Delta_{out} \approx \left[ uQ(z^*) - Q(z^*) + 1 \right] \Delta_{in} \\
+ \frac{1}{C} \left[ L_p Q(z^*) - (1 - Q(z^*))\bar{q} \right] \\
\] (9)

Now, (9) is approximately equal to (6).

**C. Aggregated ON-OFF cross traffic**

In this section, instead of the Poisson cross traffic, we consider the aggregated ON-OFF traffic model [20] to incorporate the self-similarity of the Internet traffic. The fractional Brownian motion (FBM) with parameter $\alpha$ [20] is described as follows:

For any $t \geq 0$ and $h > 0$, the increment $X(t + h) - X(t)$ of FBM $X(t)$ has the normal distribution with the mean zero and the variance $h^{2\alpha}$. 
Now, we briefly explain the ON-OFF model. The ON-OFF model has $N$ independent traffic sources $X_i(t), t \in \mathbb{Z}, i = 1, \cdots, N$ where each source is a reward renewal process with independent and identically distributed ON and OFF periods [20]. Then, we can consider the cumulative process $Y_N(Tt)$ as

$$Y_N(Tt) = \int_0^{Tt} \left( \sum_{i=1}^N X_i(s) \right) ds,$$

where $T$ is a constant. Thus, $Y_N(Tt)$ measures the total traffic up to time $Tt$. The behavior of $Y_N(Tt)$ for large $T$ and $N$ is given as the following theorem in [20].

**Theorem [20] (ON-OFF model and FBM)** $Y_N(Tt)$ behaves statistically as

$$\frac{\mathbb{E}[\tau_{on}]}{\mathbb{E}[\tau_{on}] + \mathbb{E}[\tau_{off}]} N T t + K N^{1/2} T^H B_H(t)$$

for large $T$ and $N$ where $H = (3 - \alpha)/2$, $B_H(t)$ is FBM with parameter $H$, and $K > 0$ is a quantity depending only on the distributions of $\tau_{on}$ and $\tau_{off}$.

The difference of FBM from the Poisson cross traffic is the relation between the variance $\sigma^2_{Y_N(Tt)}$ and the time interval $Tt$. From the above theorem, we have $\sigma^2_{Y_N(Tt)} = K \sigma^2 |Tt|^{2H}$, $\sigma^2 = \mathbb{E}[X(t)^2] - (\mathbb{E}[X(t)])^2$, and $\frac{1}{2} < H < 1$ for long-range dependent traffic. Since the resulting $Y_N(Tt)$ can be considered as a non-stationary Gaussian process with variance $\sigma^2_{Y_N(Tt)}$, in the time interval of $\Delta_{in}$, we have $\sigma^2_{Y_N(\Delta_{in})} \propto \Delta_{in}^{2H} = \Delta_{in}^{3-\alpha}$, $1 < \alpha < 2$, while we have $\sigma^2_{Y_N(\Delta_{in})} \propto \Delta_{in}$ for the Poisson cross traffic, which corresponds to the case of $H = 1/2$ or $\alpha = 2$. The standard deviation of the normalized traffic volume arrived in the time interval of $\Delta_{in}$ is proportional to $\frac{1}{\sqrt{\Delta_{in}}}$ in the Poisson cross traffic while $\frac{1}{(\Delta_{in})^{1-H}}$ in the FBM cross traffic with $\frac{1}{2} < H < 1$. Hence, as the observation interval increases, i.e., $\Delta_{in} \to \infty$, the variance of the normalized FBM decreases more slowly than the Poisson traffic, which is an intrinsic feature of the self-similar traffic [20]. We will show the effect of this feature on the model (6) via ns-2 simulations in Section V.
IV. Stochastic packet-pair model: Multi-hop case

Here, we derive a model for a multi-hop path which has a single tight link. Consider \( M \) links on an end-to-end path as in Fig. 2. Let \( C_i, X_i, \) and \( q_i \) denote, respectively, the link capacity, the amount of cross traffic, and the queue length on the \( i \)-th link, \( 1 \leq i \leq M \). Also, the average packet sending rate of \( X_i \) and the link utilization of link \( i \) are denoted by \( \lambda_i \) and \( u_i \), respectively. Furthermore, let \( \Delta_i \) denote the inter-departure time from the \( i \)-th queue. Then \( \Delta_i \) becomes the inter-arrival time at the \((i + 1)\)-th queue, and \( \Delta_{in} = \Delta_0 \) and \( \Delta_{out} = \Delta_M \). In this section, for simplicity, we only consider Poisson cross traffic with the same packet size for each link. Note that other kind of cross traffic can be considered in a similar way.

A. Two-hop case

Before dealing with a general multi-hop path, we first consider a two-hop path, i.e., \( M = 2 \) in Fig. 2. First, we assume that \( \Delta_1^* < \Delta_2^* \). This implies that the available bandwidth of the path is that of the second link. From (6), we have

\[
\bar{\Delta}_1 = \left( u_1 Q(x_{k_1^*} - 1) - Q(x_{k_1^*}) + 1 \right) \Delta_0 + \frac{1}{C_1} \left( L_p Q(x_{k_1^*}) - (1 - Q(x_{k_1^*}))q_1 \right).
\]

Here, \( x_{k_1^*} = \frac{k_1^*-\lambda_1\Delta_{in}}{\sqrt{\lambda_1\Delta_{in}}} \) where \( k_i^* = \max(0, \lfloor (C_i \Delta_{i-1} - L_p)/L_0 \rfloor) \), \( i = 1, \cdots, M \).

Similarly,

\[
\bar{\Delta}_2 = \mathbb{E}[\Delta_2] = \mathbb{E}_{X_1} \left[ \mathbb{E}_{X_2|X_1}[\Delta_2|\Delta_1] \right] = \mathbb{E}_{X_1} \left[ \left( u_2 Q(x_{k_2^*} - 1) - Q(x_{k_2^*}) + 1 \right) \Delta_1 + \frac{1}{C_2} \left( L_p Q(x_{k_2^*}) - (1 - Q(x_{k_2^*}))q_2 \right) \right].
\]

(10)

Now, we derive an explicit relationship between \( \bar{\Delta}_2 \) and \( \Delta_0 \) under the assumption of a single tight link. If we further assume that \( \Delta_1^* \ll \Delta_2^* \) and \( \Delta_0 \) is around \( \Delta_2^* \), then \( \Delta_0 \gg \Delta_1^* \). We
have $\Delta_1 \approx \Delta_0 - \overline{q}_1/C_1$ from (6). In this case, we have

$$
\overline{\Delta}_2 \approx \left( u_2 Q(x_{k_2}^{*}-1) - Q(x_{k_2}^{*}) + 1 \right) \left( \Delta_0 - \overline{q}_1 \right) \\
+ \frac{1}{C_1} \left( L_p Q(x_{k_2}^{*}) - (1 - Q(x_{k_2}^{*})) \overline{q}_2 \right).
$$

(11)

In a similar manner, when $\Delta_0$ is around $\Delta_1$, we have

$$
\overline{\Delta}_2 \approx \left( u_1 Q(x_{k_1}^{*}-1) - Q(x_{k_1}^{*}) + 1 \right) \Delta_0 \\
+ \frac{1}{C_1} \left( L_p Q(x_{k_1}^{*}) - (1 - Q(x_{k_1}^{*})) \overline{q}_1 \right).
$$

(12)

Note that, in finding the available bandwidth along the path, only (11) is needed. Also, note that (11) and (12) are valid under the assumption of $\Delta_*^1 \ll \Delta_*^2$. As $\Delta_*^1$ approaches $\Delta_*^2$, i.e., the available bandwidth of the first link approaches that of the second link, the assumption is no longer valid, and (11) and (12) become inaccurate.

Now, we consider the case of $\Delta_*^1 > \Delta_*^2$, i.e., the available bandwidth of the path is that of the first link. Similar to the case of $\Delta_*^1 < \Delta_*^2$, we further assume $\Delta_*^1 \gg \Delta_*^2$. When $\Delta_0$ is around $\Delta_*^1$, we obtain

$$
\overline{\Delta}_2 \approx \left( u_1 Q(x_{k_1}^{*}-1) - Q(x_{k_1}^{*}) + 1 \right) \Delta_0 \\
+ \frac{1}{C_1} \left( L_p Q(x_{k_1}^{*}) - (1 - Q(x_{k_1}^{*})) \overline{q}_1 \right) - \overline{q}_2 \frac{C_2}{C_1}.
$$

(13)

Hence, it is sufficient to analyze the behavior of the packet pair technique in the vicinity of $\Delta_{in} = \Delta_*^s$ where $s := \arg \max_i \Delta_*^i$.

**B. Multi-hop case**

Since we are primarily concerned about the way of obtaining information on the available bandwidth of an end-to-end path, it is sufficient to study the behavior of the packet pair technique in the vicinity of $\Delta_{in} = \Delta_*^s$ where the $s$-th link has the minimum available bandwidth. If we assume that $\Delta_*^s \gg \Delta_*^i, \ i = 1, \cdot \cdot \cdot , M, \ i \neq s$, as the cases of (11) and
(13), we can derive the following approximate model for the multi-hop path:

\[
\Delta_{out} = \left( u_s Q(x_{k_s^*} - 1) - Q(x_{k_s^*}) + 1 \right) \Delta_{s-1} \\
+ \frac{1}{C_s} \left( L_p Q(x_{k_s^*}) - (1 - Q(x_{k_s^*}))\bar{q}_s \right) \\
- \sum_{i=s+1}^{M} \frac{\bar{q}_i}{C_i},
\]

(14)

where \( \Delta_{s-1} = \Delta_{in} - \sum_{i=1}^{s-1} \frac{\bar{q}_i}{C_i}, \) \( x_{k_s^*} = \frac{k_s^* - \lambda_s \Delta_{s-1}}{\sqrt{\lambda_s \Delta_{s-1}}}, \) and \( k_s^* = \max(0, [(C_s \Delta_{s-1} - L)/L_0]) \), \( i = 1, \ldots, M. \)

The multi-hop model (14) shows that the available bandwidth of any link after the tight link is not observable in an end-to-end manner. By using the packet pair technique, we can obtain the information only on the available bandwidth of the \( i \)-th link where \( i \leq s. \)

V. Model validation

In this section, we validate our stochastic model via ns-2 simulations and experimental results.

A. ns-2 simulations for single-hop topology

We perform simulations for a single-hop topology composed of two drop-tail routers that are connected via a link with the capacity of 1Mb/s and the propagation delay of 15ms. First, we investigate the deterministic model (1). In the simulation, the cross traffic consists of \( N \) Pareto sources, and each source generates packets at the rate of 32 Kb/s with the packet size of \( L_0 = 50 \) bytes. Figure 3 (a) and (b) shows \( \Delta_{in} - \overline{\Delta_{out}} \) versus \( \Delta_{in} \) for \( N = 7 \) and 15, respectively. The sizes of probing packets are 500, 1000, and 1500 bytes, respectively. Note that we introduce \( \Delta_{in} - \overline{\Delta_{out}} =: \Lambda_{out} \) to highlight the change of \( \frac{\partial \overline{\Delta_{out}}}{\partial \Delta_{in}} \) around the characteristic value \( \Delta^* (= (L_p + q)/(C - r)) \). Each data point is an average value of 300 trials. The solid lines are obtained from the deterministic model (1) with \( q = 0. \) We can see that the simulation data asymptotically converges to the analytical values. However,
the model error around $\Delta_{in} = \Delta^*$ becomes larger as the link utilization and/or the probing packet size $L_p$ increases. $\frac{\partial \Lambda_{out}}{\partial \Delta_{in}}$ changes smoothly around $\Delta_{in} = \Delta^*$ in the simulation results while it changes abruptly at $\Delta_{in} = \Delta^*$ for the deterministic model. This indicates that the stochastic nature of cross traffic makes it difficult to find the value of $\Delta^*$ in practice.

Now, we validate the stochastic model (6). Figures 4, 5, and 6 show $\Lambda_{out}$ versus $\Delta_{in}$ for the Poisson, the exponential ON-OFF, and the Pareto ON-OFF cross traffic, respectively. Each figure shows the result for different sizes of probing packets ($L_p = 500, 1000,$ and $1500$ bytes) with the utilization of $24\%$ and $50\%$, respectively. In all the simulations, we use $L_0 = 50$ bytes for the cross traffic packet size. In Fig. 4, we can see that each of the simulation results agrees very well with the stochastic model (6). This is because the Poisson sources are used both in the model and the simulations. Now, we consider the exponential ON-OFF sources in Fig. 5. In Fig. 5, the Poisson sources are used for the model as in Fig. 4 while the exponential ON-OFF sources are employed in the simulation. Accordingly, we can see that there exist some errors between the model and the simulation data. Note that this error comes from the discrepancy between the Poisson and the exponential ON-OFF traffic and can be reduced if we know the variance of the exponential ON-OFF sources. Since the variance of the exponential ON-OFF traffic is larger than that of the Poisson traffic, the slope of $\Lambda_{out}$ changes more slowly around $\Delta_{in} = \Delta^*$ in the simulation data than in the model. Next, we consider the case of the Pareto ON-OFF sources in Fig. 6. Similarly as in Fig. 5, Fig. 6 shows some error around $\Delta_{in} = \Delta^*$ for the same reason. Note that the variance of the Pareto ON-OFF traffic is even larger than that of the exponential ON-OFF traffic, and $\Lambda_{out}$ changes more slowly in the simulation data of Fig. 6 than that of Fig. 5.

Now, we vary the packet size of the Poisson cross traffic to investigate the effect of the packet size on the model accuracy. In developing our model in Section 3 and Section 4, we have ignored the round-off effect in $k^* = [(C\Delta_{in} - L_p)/L_0]$, which may be a problem with a small value of $k^*$. In the vicinity of $\Delta_{in} = \Delta^*$, $k^*$ is proportional to $L_p/L_0$. Hence, we
need to see the effect of the packet size $L_0$ in cross traffic on the model accuracy. Figure 7 (a) and (b) show the simulation results of $\Lambda_{\text{out}}$ vs. $\Delta_{\text{in}}$ under different link utilizations for the packet sizes of 50 and 250 bytes, respectively. When the packet size is 50 bytes, we can verify from Fig. 7 (a) that the proposed model is quite accurate. When the packet size is 250 bytes in Fig. 7 (b), there exists some error between the model and the simulation data. Still, the overall trend of the graphs is quite similar. We will also investigate the effect of the packet size in cross traffic with the empirical results in Section 5-C.

B. ns-2 simulations for two-hop topology

We now validate the stochastic model for the two-hop topology, which is composed of three drop-tail routers that are connected via two links with capacity of either $C_1 = 2$ Mb/s and $C_2 = 1$ Mb/s (which corresponds to the case $\Delta_1^* < \Delta_2^*$) or $C_1 = 1$ Mb/s and $C_2 = 2$ Mb/s (which corresponds to the case $\Delta_1^* > \Delta_2^*$). The cross traffic on the first link (the second link) is composed of 23 (15) Pareto sources that give an aggregate rate of $r = 736$ Kbps ($r = 480$ Kbps). The packet size $L_0$ of cross traffic is 100 bytes. Figure 8 (a) and (b) shows the analytical and simulation results for the cases of $\Delta_1^* < \Delta_2^*$ and $\Delta_1^* > \Delta_2^*$, respectively. In the case of $\Delta_1^* < \Delta_2^*$ ($L_p = 1500$ bytes, $C_1 = 2$ Mb/s, $C_2 = 1$ Mb/s), we can see two changes in $\frac{d\Lambda_{\text{out}}}{d\Delta_{\text{in}}}$ around $\Delta_{\text{in}} = \Delta_1^* = 9.5$ ms and $\Delta_{\text{in}} = \Delta_2^* = 23.1$ ms in Fig. 8. Note that $\Delta_2^*$ gives the information on the available bandwidth along the path. In the case of $\Delta_1^* > \Delta_2^*$ ($L_p = 1500$ bytes, $C_1 = 1$ Mb/s, $C_2 = 2$ Mb/s), we only see one change in $\frac{d\Lambda_{\text{out}}}{d\Delta_{\text{in}}}$ around $\Delta_{\text{in}} = \Delta_1^* = 23.1$ ms. From Fig. 8, we can see that the approximate multi-hop model (14) agrees quite well around the largest characteristic value, which gives information on the available bandwidth along the path.

C. Empirical results on a two-hop network

Here, we investigate the proposed stochastic model under real network environment. The network is composed of three routers $R_1$, $R_2$, and $R_3$ that are connected via two
100 Mb/s links and five hosts, denoted by $l_1$, $l_2$, and $H_i$, $i = 1, \cdots, 5$, respectively, as depicted in Fig. 9. Here, each host is a PC with a single 2.66 GHz processor and 1 GByte RAM that runs Redhat Linux Release 9.0 with kernel version 2.4.20-31.9 and the routers used in the experiments are Cisco 1700 Series. The path we injected the probing packets was $P : H_1 \to R_1 \to R_2 \to R_3 \to H_3$, which constitutes a two-hop path. Each of the experimental data in Fig. 10 – Fig. 12 is an average of 5 trials.

First, we conducted experiments when there was no cross traffic. In this case, both of the deterministic and stochastic models give

$$
\Delta_{\text{out}} = \begin{cases} 
\frac{L_p}{C}, & \Delta_{\text{in}} \leq \Delta^* \left( = \frac{L_p + \bar{q}}{C} \right); \\
\Delta_{\text{in}} - \frac{\bar{q}}{C}, & \text{otherwise.} 
\end{cases}
$$

(15)

Hence, we can obtain the link capacity by observing either the critical value $\Delta^* (= \frac{L_p}{C})$ or the slope of the line segment when $\Delta_{\text{in}} \leq \Delta^*$. Analytical and experimental results with no cross traffic are shown in Fig. 10. From the figure, we can notice that $\frac{\bar{q}}{C}$ in the experimental results is approximately 20 $\mu$sec. Hence, we use this value of $\frac{\bar{q}}{C}$ for the analytical results.

The reason is that $\bar{q}$ in real environment is not easy to obtain analytically since $\bar{q}$ depends on the queueing policy at each router. In Fig. 10, we see that (15) agrees with the experimental results very well.

Now, the Poisson cross traffic is generated and traverses the path from $H_5$ to $H_4$. Hence, the link $l_2 : R_2 \to R_3$ becomes the tight link of the path $P$. Figure 11 (a) and (b) shows $\Lambda_{\text{out}}$ for $L_p = 500$, 1000, and 1500 bytes, when $L_0$ is 250 bytes and the link utilization is 25% and 50%, respectively. We can see that the stochastic model agrees quite well with the experimental data in the figure. The path $P$ used in the experiments was composed of two links $l_1 : R_1 \to R_2$ and $l_2 : R_2 \to R_3$, and the corresponding characteristic values of the link $l_1$ are $\Delta_1^* = (L_p + \bar{q}_1)/C_1 = 40, 80, \text{and} 120 \mu s$ for $L_p = 500, 1000, \text{and} 1500$ bytes, respectively. Similarly, the characteristic values of link $l_2$ when $u = 25\% \ (u = 50\%)$ are $\Delta_2^* = (L_p + \bar{q}_2)/(C_2 - r_2) = 80, 400/3$, and $560/3 \mu s$ $(120, 200, \text{and} 280 \mu s)$ for $L_p = 500,$
1000, and 1500 bytes, respectively. Since $\Delta_1^* < \Delta_2^*$, this corresponds to the case in which both of $\Delta_1^*$ and $\Delta_2^*$ are observed as in Fig. 8 (a). Note that the experimental data changes smoothly around $\Delta_{in} = \Delta_2^*$ as expected, and this makes it difficult to find $\Delta_2^*$. This difficulty is not expected with the deterministic model (1). Figure 12 shows the results of the similar experiments with $L_0 = 750$ bytes. From the figure, we can see that the stochastic model follows the experimental data quite well.

VI. Discussion

In this section, based on the proposed model, we discuss the effect of stochastic cross traffic in available bandwidth estimation. We point out that the stochastic nature of cross traffic can introduce significant error in available bandwidth estimation algorithms that try to find out the characteristic value.

A. Effect of stochastic cross traffic on the available bandwidth estimation

From the analysis in Section 3 and Section 4, $\Delta_{out}$ is a stochastic process that is characterized by $C$, $\Delta_{in}$, $L_p$, $L_0$, and $X(\Delta_{in})$ where $C = (C_l, l \in L(P))$, $X(\Delta_{in}) = (X_l(\Delta_{in}), l \in L(P))$, and $L(P)$ is the set of links that constitutes the path $P$. Note that the stochastic behaviors of $\Delta_{out}$ comes from the stochastic nature of $X(\Delta_{in})$. In the packet pair technique for available bandwidth measurement, we observe $\Delta_{out}$ with the knowledge of $\Delta_{in}$ and $L_p$ and try to find out the value of the available bandwidth on the end-to-end path. The difficulty of bandwidth measurement via the packet-pair technique lies in the randomness of $\Delta_{out}$: A single sample of $\Delta_{out}$ does not have any information on the available bandwidth in practice. Hence, we need a statistical measure that can be exploited for obtaining the value of the available bandwidth. A natural one is the expectation of $\Delta_{out}$, i.e., $\overline{\Delta}_{out}$, or equivalently, $\Lambda_{out} (= \Delta_{in} - \overline{\Delta}_{out})$. From Section III, $\Lambda_{out}$ depends on several variables, i.e.,

$$\Lambda_{out} = F(\Delta_{in}, C, L_p; f_X(\Delta_{in}), L_0)$$

where $f_X(\Delta_{in}) = \{f_X(\Delta_{in}), l \in L(P)\}$ and $f_X(\Delta_{in})$ is the pdf of cross traffic $X_l(\Delta_{in})$ at each link.
Now, we further look into the relationship between $\Lambda_{out}$ and $\Delta_{in}$ for several cases of cross traffic. First, consider the case of the Gaussian cross traffic. The Gaussian cross traffic is fully characterized by the mean and the variance of the process, and we have $\Lambda_{out} = F(\Delta_{in}, C, L_p; m(\Delta_{in}), \sigma(\Delta_{in}), L_0)$, where $m(\Delta_{in}) = \{m_l(\Delta_{in}), l \in L(P)\}$ and $\sigma(\Delta_{in}) = \{\sigma_l(\Delta_{in}), l \in L(P)\}$. Here, $m_l(\Delta_{in})$ and $\sigma_l(\Delta_{in})$ represent the mean and the standard deviation of $X_l(\Delta_{in})$, respectively. If cross traffic is the FBM, we have $m(\Delta_{in}) = r(\Delta_{in}) = \{r_l(\Delta_{in}), l \in L(P)\}$ and $\sigma(\Delta_{in}) = K(\Delta_{in}) = \{K_l(\Delta_{in})\}$, where $r_l$ is the average traffic rate at link $l$, and $K_l$ is a scaling constant for the standard deviation of $X_l(\Delta_{in})$. Hence, $\Lambda_{out} = F(\Delta_{in}, C, L_p; r(\Delta_{in}), K(\Delta_{in}), L_0)$.

The Poisson cross traffic corresponds to the case of $\alpha = 2$ in the FBM cross traffic with $\sigma(\Delta_{in}) = r(\Delta_{in})$. Hence, $\Lambda_{out} = F(\Delta_{in}, C, L_p; r(\Delta_{in}), r(\Delta_{in}), L_0)$. Now, we further investigate the case of the Poisson cross traffic in a single hop topology. From the result in Section III. A, we have

$$\Lambda_{out} \approx \left( Q(x_{k^*}) - uQ(x_{k^*-1}) \right) \Delta_{in} \quad - \frac{1}{C} \left( L_p Q(x_{k^*}) - (1 - Q(x_{k^*})) \right)$$

$$= : F(\Delta_{in}, C, L_p; \lambda \Delta_{in}, \lambda \Delta_{in}, L_0), \quad (16)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\xi^2/2} d\xi$ and $x_{k^*} = \frac{(C/L_0 - \lambda)\Delta_{in} - L_p/L_0}{\sqrt{\lambda \Delta_{in}}}$.

Among the parameters of $F(\cdot)$, $C$ is fixed for a given path $P$, and $r$ and $L_0$ are the characteristics of cross traffic that we have no control. $\Delta_{in}$ and $L_p$ are the only parameters of which we have control.

In summary, $\Lambda_{out}$ can be considered as a function with two control variables $\Delta_{in}$ and $L_p$ for a given pdf of cross traffic and link capacities. Hence, our investigation gives a new insight on available bandwidth estimation, i.e., not only the relationship between $\Lambda_{out}$ and $\Delta_{in}$, but also the relationship between $\Lambda_{out}$ and $L_p$ can be exploited to obtain an efficient available bandwidth measurement algorithm. Note that most of the recent tools for the available bandwidth measurement pay attention only to the relationship between $\Lambda_{out}$ and
\( \Delta_{in} \). As \( \frac{\partial \Delta_{out}}{\partial \Delta_{in}} \) changes abruptly at \( \Delta_{in} = \Delta^* \) (= \( L/A \)) in the deterministic framework, most of the measurement methods for the available bandwidth try to find \( \Delta_{in} = \Delta^* \) by observing the change of \( \frac{\partial \Delta_{out}}{\partial \Delta_{in}} \) [2] [10] [11] [12] [13]. However, we have shown that \( \frac{\partial \Delta_{out}}{\partial \Delta_{in}} \) does not change abruptly at \( \Delta_{in} = \Delta^* \), but smoothly around \( \Delta_{in} = \Delta^* \) in the stochastic framework.

The difference between the deterministic model and the stochastic model is illustrated in Fig. 13. From the figure, we can see that available bandwidth measurement algorithms that try to find out the characteristic value \( \Delta^* \) in \((\Delta_{in}, \Delta_{out})\) curve may find an inaccurate value \( \hat{\Delta}^* \) as in Fig. 13 due to the smooth change of the curve. Actually, there exists a region around \( \Delta^* \) that can be considered as neither under-utilized nor fully-utilized. This region, termed as the biased probing region in [14], results from the stochastic nature of cross traffic. The biased probing region makes it challenging to design an accurate available bandwidth measurement algorithm.

B. Discussion on recent available bandwidth measurement mechanisms

Here, we investigate two recent end-to-end available bandwidth measurement methods, i.e., pathload [2] and IGI [11] in relation with the proposed stochastic model. The stochastic model allows us to better understand the performance and limitation of each method.

B.1 Pathload

The key idea of pathload is to monitor variations in the one way delays (OWD) of the probing packets [2]. The source sends a number of equal-sized packets, i.e., a “periodic packet stream” to the receiver at a certain rate \( R \). If the stream rate \( R \) is smaller than the available bandwidth, the probing packets will not cause congestion at the tight link and the OWDs will not increase. If \( R \) is larger than the available bandwidth, then the OWDs will increase linearly. However, the OWDs of a stream do not show an increasing or non-increasing trend clearly in some cases, which is termed as the grey region [2]. This can be explained by the stochastic framework. When there are \( N + 1 \) packets equally spaced in
a packet stream, sending a packet stream can be considered as $N$ Bernoulli trials, each of which has two events, i.e., either OWD increases or does not. Since $\Delta_{in}$ is the time difference between two consecutive packets in a packet train, the link is fully-utilized with $P(\Gamma)$ and under-utilized with $P(\Gamma^c) = 1 - P(\Gamma)$. The OWD increases if the link is fully-utilized and decreases otherwise. Hence, the OWDs of a stream follows a binomial distribution with $p = P(\Gamma)$, and the number of increasing OWDs is $Np$ on average. For example, if $\Delta_{in} = \Delta^*$, $P(\Gamma) = Q(x_k^*)|_{\Delta_{in} = \Delta^*} = 0.5$ for the Poisson cross traffic, and consequently, half of the OWDs increase and the others do not increase on average. Therefore, it will not be easy to observe the trend of the OWDs. Note that the grey region results from the stochastic nature of cross traffic, and expands as the variance of cross traffic increases.

B.2 IGI

The Initial Gap Increasing (IGI) algorithm is based on packet trains to capture the average behavior of the cross traffic [11]. As already explained in the previous section, IGI is one of those methods that seek the critical value $\Delta^c$. Here, we explain how IGI works with stochastic cross traffic. In the stochastic sense, IGI calculates the following quantity $\hat{r}$ for estimating the traffic load $r$:

$$\hat{r} = \frac{E[Y(\Delta_{in})]}{E[\Delta_{out}]} \bigg|_{\Delta_{in} = E[\Delta_{out}]}$$

where

$$Y(\Delta_{in}) = \begin{cases} X(\Delta_{in}), & \text{if } A_k \in \Gamma; \\ 0, & \text{otherwise}. \end{cases}$$

From Section III. A. 2, we have

$$E[Y(\Delta_{in})] = \int_{x^*}^{\infty} x f_X(x) dx,$$

where $x^* = \max(0, [C\Delta_{in} - L_p])$. 
Hence, we can rewrite the above equation with the critical value $\Delta^c$ as follows:

$$\hat{r} = \frac{\int_{x^*}^\infty x f_X(x) \, dx}{\Delta_{in}} \bigg|_{\Delta_{in} = \Delta^c}.$$ 

In practice, IGI finds $\Delta^c$ by gradually increasing $\Delta_{in}$ starting from a small value. With the fluid cross traffic with a constant rate, $\int_{x^*}^\infty x f_X(x) \, dx = m_X = r \Delta_{in}$, and consequently, we have $\hat{r} = r$. However, with stochastic cross traffic, $\int_{x^*}^\infty x f_X(x) \, dx$ will be smaller than $r \Delta_{in}$. In this case, we have $\hat{r} < r$, and consequently, the available bandwidth will be overestimated.

VII. Conclusion

In this paper, we have derived an explicit packet pair model, which reflects the stochastic nature of cross traffic. We have investigated the mathematical relationship between the input and the output gaps of a packet pair in a single-hop case under the reasonable assumption of stationary Poisson cross traffic. We also have derived an explicit multi-hop model for the case of a single tight link. We have shown that the stochastic model matches well with the \texttt{ns-2} simulation results and the experimental results. Based on the stochastic model, we have pointed out that it is difficult to find the characteristic value, which is due to the stochastic nature of cross traffic. The stochastic model will be helpful in finding a good mechanism for estimating the available bandwidth.

We need to enhance the stochastic model in several aspects. In developing a multi-hop model in our analysis, we have made an assumption of a single tight link. However, this assumption will be unrealistic if any two or more links have nearly the same available bandwidths. Hence, development of an extended stochastic model without this assumption is remained as future work. We also need to verify the proposed model with the real Internet traffic to see the effect of the assumption of the stationary Poisson cross traffic.
References


Fig. 1. Arrival and departure of a packet pair on a single hop.

(a) Case when the second probing packet sees the server busy   (b) Case when the second probing packet sees the server idle

Fig. 2. Multi-hop model.
Fig. 3. Validation of the deterministic model: $\Delta_{\text{in}} - \Delta_{\text{out}} (= \Lambda_{\text{out}})$ versus $\Delta_{\text{in}}$.

Fig. 4. $\Lambda_{\text{out}}$ versus $\Delta_{\text{in}}$ for the Poisson cross traffic in a single-hop topology.

Fig. 5. $\Lambda_{\text{out}}$ versus $\Delta_{\text{in}}$ for the exponential ON-OFF cross traffic in a single-hop topology.
Fig. 6. $\Lambda_{out}$ versus $\Delta_{in}$ for the Pareto ON-OFF cross traffic in a single-hop topology.

(a) Utilization = 24%

(b) Utilization = 50%

Fig. 7. Effect of the packet size of cross traffic on model accuracy ($L_p = 1500$ bytes).

(a) $L_0 = 50$ bytes

(b) $L_0 = 250$ bytes
Fig. 8. $\Lambda_{out}$ versus $\Delta_{in}$ for the Pareto ON-OFF cross traffic in the two link case.

(a) When $\Delta^*_1 < \Delta^*_2$: Both $\Delta^*_1$ and $\Delta^*_2$ are observable

(b) When $\Delta^*_1 > \Delta^*_2$: $\Delta^*_2$ is not observable

Fig. 9. The topology used for the experiments.

Fig. 10. $\Lambda_{out}$ versus $\Delta_{in}$ with no cross traffic in the two-hop experiment.
Fig. 11. $\Lambda_{\text{out}}$ versus $\Delta_{\text{in}}$ with the Poisson cross traffic of $L_0 = 250$ bytes in the two-hop experiment.

Fig. 12. $\Lambda_{\text{out}}$ versus $\Delta_{\text{in}}$ with the Poisson cross traffic of $L_0 = 750$ bytes in the two-hop experiment.

Fig. 13. Effect of stochastic cross traffic on the estimation of the characteristic value $\Delta^*$. 